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Diploma Programme

Mathematics: analysis and approaches formula booklet

For use during the course and in the examinations
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Prior learning – SL and HL

Area of a parallelogram	$A = bh$, where b is the base, h is the height
Area of a triangle	$A = \frac{1}{2}(bh)$, where b is the base, h is the height
Area of a trapezoid	$A = \frac{1}{2}(a + b)h$, where a and b are the parallel sides, h is the height
Area of a circle	$A = \pi r^2$, where r is the radius
Circumference of a circle	$C = 2\pi r$, where r is the radius
Volume of a cuboid	$V = lwh$, where l is the length, w is the width, h is the height
Volume of a cylinder	$V = \pi r^2 h$, where r is the radius, h is the height
Volume of a prism	$V = Ah$, where A is the area of cross-section, h is the height
Area of the curved surface of a cylinder	$A = 2\pi r h$, where r is the radius, h is the height
Distance between two points (x_1, y_1) and (x_2, y_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Coordinates of the midpoint of a line segment with endpoints (x_1, y_1) and (x_2, y_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

Topic 1: Number and algebra – SL and HL

SL 1.2	<p>The nth term of an arithmetic sequence</p> <p>The sum of n terms of an arithmetic sequence</p>	$u_n = u_1 + (n - 1)d$ $S_n = \frac{n}{2}(2u_1 + (n - 1)d); S_n = \frac{n}{2}(u_1 + u_n)$
SL 1.3	<p>The nth term of a geometric sequence</p> <p>The sum of n terms of a finite geometric sequence</p>	$u_n = u_1 r^{n-1}$ $S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, r \neq 1$
SL 1.4	Compound interest	$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$ <p>where FV is the future value, PV is the present value, n is the number of years, k is the number of compounding periods per year, $r\%$ is the nominal annual rate of interest</p>
SL 1.5	Exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a b, \text{ where } a > 0, b > 0, a \neq 1$
SL 1.7	Exponents and logarithms	$\log_a xy = \log_a x + \log_a y$ $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^m = m \log_a x$ $\log_a x = \frac{\log_b x}{\log_b a}$
SL 1.8	The sum of an infinite geometric sequence	$S_\infty = \frac{u_1}{1 - r}, r < 1$
SL 1.9	Binomial theorem $n \in \mathbb{N}$	$(a + b)^n = a^n + {}^n C_1 a^{n-1} b + \dots + {}^n C_r a^{n-r} b^r + \dots + b^n$ ${}^n C_r = \frac{n!}{r!(n-r)!}$

Topic 1: Number and algebra – HL only

AHL 1.10	<p>Combinations</p> ${}^n C_r = \frac{n!}{r!(n-r)!}$ <p>Permutations</p> ${}^n P_r = \frac{n!}{(n-r)!}$ <p>Extension of binomial theorem, $n \in \mathbb{Q}$</p> $(a+b)^n = a^n \left(1 + n \left(\frac{b}{a} \right) + \frac{n(n-1)}{2!} \left(\frac{b}{a} \right)^2 + \dots \right)$
AHL 1.12	<p>Complex numbers</p> $z = a + bi$
AHL 1.13	<p>Modulus-argument (polar) and exponential (Euler) form</p> $z = r(\cos \theta + i \sin \theta) = re^{i\theta} = r \operatorname{cis} \theta$
AHL 1.14	<p>De Moivre's theorem</p> $[r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta) = r^n e^{in\theta} = r^n \operatorname{cis} n\theta$

Topic 2: Functions – SL and HL

SL 2.1	Equations of a straight line	$y = mx + c$; $ax + by + d = 0$; $y - y_1 = m(x - x_1)$
	Gradient formula	$m = \frac{y_2 - y_1}{x_2 - x_1}$
SL 2.6	Axis of symmetry of the graph of a quadratic function	$f(x) = ax^2 + bx + c \Rightarrow$ axis of symmetry is $x = -\frac{b}{2a}$
SL 2.7	Solutions of a quadratic equation	$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, a \neq 0$
	Discriminant	$\Delta = b^2 - 4ac$
SL 2.9	Exponential and logarithmic functions	$a^x = e^{x \ln a}$; $\log_a a^x = x = a^{\log_a x}$ where $a, x > 0, a \neq 1$

Topic 2: Functions – HL only

AHL 2.12	Sum and product of the roots of polynomial equations of the form $\sum_{r=0}^n a_r x^r = 0$	Sum is $-\frac{a_{n-1}}{a_n}$; product is $\frac{(-1)^n a_0}{a_n}$
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Topic 3: Geometry and trigonometry – SL and HL

SL 3.1	<p>Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2)</p> <p>Coordinates of the midpoint of a line segment with endpoints (x_1, y_1, z_1) and (x_2, y_2, z_2)</p> <p>Volume of a right-pyramid</p> <p>Volume of a right cone</p> <p>Area of the curved surface of a cone</p> <p>Volume of a sphere</p> <p>Surface area of a sphere</p>	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$ $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$ $V = \frac{1}{3} Ah, \text{ where } A \text{ is the area of the base, } h \text{ is the height}$ $V = \frac{1}{3} \pi r^2 h, \text{ where } r \text{ is the radius, } h \text{ is the height}$ $A = \pi r l, \text{ where } r \text{ is the radius, } l \text{ is the slant height}$ $V = \frac{4}{3} \pi r^3, \text{ where } r \text{ is the radius}$ $A = 4\pi r^2, \text{ where } r \text{ is the radius}$
SL 3.2	<p>Sine rule</p> <p>Cosine rule</p> <p>Area of a triangle</p>	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $c^2 = a^2 + b^2 - 2ab \cos C; \cos C = \frac{a^2 + b^2 - c^2}{2ab}$ $A = \frac{1}{2} ab \sin C$
SL 3.4	<p>Length of an arc</p> <p>Area of a sector</p>	$l = r\theta, \text{ where } r \text{ is the radius, } \theta \text{ is the angle measured in radians}$ $A = \frac{1}{2} r^2 \theta, \text{ where } r \text{ is the radius, } \theta \text{ is the angle measured in radians}$

SL 3.5	Identity for $\tan \theta$	$\tan \theta = \frac{\sin \theta}{\cos \theta}$
SL 3.6	Pythagorean identity	$\cos^2 \theta + \sin^2 \theta = 1$
	Double angle identities	$\sin 2\theta = 2 \sin \theta \cos \theta$ $\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$

Topic 3: Geometry and trigonometry – HL only

AHL 3.9	Reciprocal trigonometric identities	$\sec \theta = \frac{1}{\cos \theta}$ $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$
	Pythagorean identities	$1 + \tan^2 \theta = \sec^2 \theta$ $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$
AHL 3.10	Compound angle identities	$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
	Double angle identity for tan	$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$
AHL 3.12	Magnitude of a vector	$ \mathbf{v} = \sqrt{v_1^2 + v_2^2 + v_3^2}$, where $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$

AHL 3.13	<p>Scalar product</p> <p>Angle between two vectors</p>	$\mathbf{v} \cdot \mathbf{w} = v_1 w_1 + v_2 w_2 + v_3 w_3, \text{ where } \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ $\mathbf{v} \cdot \mathbf{w} = \mathbf{v} \mathbf{w} \cos \theta, \text{ where } \theta \text{ is the angle between } \mathbf{v} \text{ and } \mathbf{w}$ $\cos \theta = \frac{v_1 w_1 + v_2 w_2 + v_3 w_3}{ \mathbf{v} \mathbf{w} }$
AHL 3.14	<p>Vector equation of a line</p> <p>Parametric form of the equation of a line</p> <p>Cartesian equations of a line</p>	$\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$ $x = x_0 + \lambda l, y = y_0 + \lambda m, z = z_0 + \lambda n$ $\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$
AHL 3.16	<p>Vector product</p> <p>Area of a parallelogram</p>	$\mathbf{v} \times \mathbf{w} = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix}, \text{ where } \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ $ \mathbf{v} \times \mathbf{w} = \mathbf{v} \mathbf{w} \sin \theta, \text{ where } \theta \text{ is the angle between } \mathbf{v} \text{ and } \mathbf{w}$ $A = \mathbf{v} \times \mathbf{w} \text{ where } \mathbf{v} \text{ and } \mathbf{w} \text{ form two adjacent sides of a parallelogram}$
AHL 3.17	<p>Vector equation of a plane</p> <p>Equation of a plane (using the normal vector)</p> <p>Cartesian equation of a plane</p>	$\mathbf{r} = \mathbf{a} + \lambda \mathbf{b} + \mu \mathbf{c}$ $\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$ $ax + by + cz = d$

Topic 4: Statistics and probability – SL and HL

SL 4.2	Interquartile range	$IQR = Q_3 - Q_1$
SL 4.3	Mean, \bar{x} , of a set of data	$\bar{x} = \frac{\sum_{i=1}^k f_i x_i}{n}, \text{ where } n = \sum_{i=1}^k f_i$
SL 4.5	Probability of an event A	$P(A) = \frac{n(A)}{n(U)}$
	Complementary events	$P(A) + P(A') = 1$
SL 4.6	Combined events	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
	Mutually exclusive events	$P(A \cup B) = P(A) + P(B)$
	Conditional probability	$P(A B) = \frac{P(A \cap B)}{P(B)}$
	Independent events	$P(A \cap B) = P(A) P(B)$
SL 4.7	Expected value of a discrete random variable X	$E(X) = \sum x P(X = x)$
SL 4.8	Binomial distribution $X \sim B(n, p)$	
	Mean	$E(X) = np$
	Variance	$\text{Var}(X) = np(1 - p)$
SL 4.12	Standardized normal variable	$z = \frac{x - \mu}{\sigma}$

Topic 4: Statistics and probability – HL only

AHL 4.13	Bayes' theorem	$P(B A) = \frac{P(B) P(A B)}{P(B) P(A B) + P(B') P(A B')}$ $P(B_i A) = \frac{P(B_i) P(A B_i)}{P(B_1) P(A B_1) + P(B_2) P(A B_2) + P(B_3) P(A B_3)}$
AHL 4.14	Variance σ^2 Standard deviation σ Linear transformation of a single random variable Expected value of a continuous random variable X Variance Variance of a discrete random variable X Variance of a continuous random variable X	$\sigma^2 = \frac{\sum_{i=1}^k f_i (x_i - \mu)^2}{n} = \frac{\sum_{i=1}^k f_i x_i^2}{n} - \mu^2$ $\sigma = \sqrt{\frac{\sum_{i=1}^k f_i (x_i - \mu)^2}{n}}$ $E(aX + b) = aE(X) + b$ $\text{Var}(aX + b) = a^2 \text{Var}(X)$ $E(X) = \mu = \int_{-\infty}^{\infty} x f(x) dx$ $\text{Var}(X) = E[(X - \mu)^2] = E(X^2) - [E(X)]^2$ $\text{Var}(X) = \sum (x - \mu)^2 P(X = x) = \sum x^2 P(X = x) - \mu^2$ $\text{Var}(X) = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx = \int_{-\infty}^{\infty} x^2 f(x) dx - \mu^2$

Topic 5: Calculus – SL and HL

SL 5.3	Derivative of x^n	$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
SL 5.5	Integral of x^n Area between a curve $y = f(x)$ and the x -axis, where $f(x) > 0$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$ $A = \int_a^b y dx$
SL 5.6	Derivative of $\sin x$ Derivative of $\cos x$ Derivative of e^x Derivative of $\ln x$ Chain rule Product rule Quotient rule	$f(x) = \sin x \Rightarrow f'(x) = \cos x$ $f(x) = \cos x \Rightarrow f'(x) = -\sin x$ $f(x) = e^x \Rightarrow f'(x) = e^x$ $f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$ $y = g(u), \text{ where } u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ $y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$ $y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
SL 5.9	Acceleration Distance travelled from t_1 to t_2 Displacement from t_1 to t_2	$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$ distance = $\int_{t_1}^{t_2} v(t) dt$ displacement = $\int_{t_1}^{t_2} v(t) dt$

SL 5.10	Standard integrals	$\int \frac{1}{x} dx = \ln x + C$ $\int \sin x dx = -\cos x + C$ $\int \cos x dx = \sin x + C$ $\int e^x dx = e^x + C$
SL 5.11	Area of region enclosed by a curve and x -axis	$A = \int_a^b y dx$

Topic 5: Calculus – HL only

AHL 5.12	Derivative of $f(x)$ from first principles	$y = f(x) \Rightarrow \frac{dy}{dx} = f'(x) = \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right)$
AHL 5.15	Standard derivatives	
	$\tan x$	$f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$
	$\sec x$	$f(x) = \sec x \Rightarrow f'(x) = \sec x \tan x$
	$\operatorname{cosec} x$	$f(x) = \operatorname{cosec} x \Rightarrow f'(x) = -\operatorname{cosec} x \cot x$
	$\cot x$	$f(x) = \cot x \Rightarrow f'(x) = -\operatorname{cosec}^2 x$
	a^x	$f(x) = a^x \Rightarrow f'(x) = a^x (\ln a)$
	$\log_a x$	$f(x) = \log_a x \Rightarrow f'(x) = \frac{1}{x \ln a}$
	$\arcsin x$	$f(x) = \arcsin x \Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}}$
	$\arccos x$	$f(x) = \arccos x \Rightarrow f'(x) = -\frac{1}{\sqrt{1-x^2}}$
	$\arctan x$	$f(x) = \arctan x \Rightarrow f'(x) = \frac{1}{1+x^2}$

AHL 5.15	Standard integrals	$\int a^x dx = \frac{1}{\ln a} a^x + C$ $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$ $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin\left(\frac{x}{a}\right) + C, \quad x < a$
AHL 5.16	Integration by parts	$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx \quad \text{or} \quad \int u dv = uv - \int v du$
AHL 5.17	Area of region enclosed by a curve and y -axis Volume of revolution about the x or y -axes	$A = \int_a^b x dy$ $V = \int_a^b \pi y^2 dx \quad \text{or} \quad V = \int_a^b \pi x^2 dy$
AHL 5.18	Euler's method Integrating factor for $y' + P(x)y = Q(x)$	$y_{n+1} = y_n + h \times f(x_n, y_n); \quad x_{n+1} = x_n + h, \quad \text{where } h \text{ is a constant (step length)}$ $e^{\int P(x) dx}$
AHL 5.19	Maclaurin series Maclaurin series for special functions	$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \dots$ $e^x = 1 + x + \frac{x^2}{2!} + \dots$ $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$ $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots$